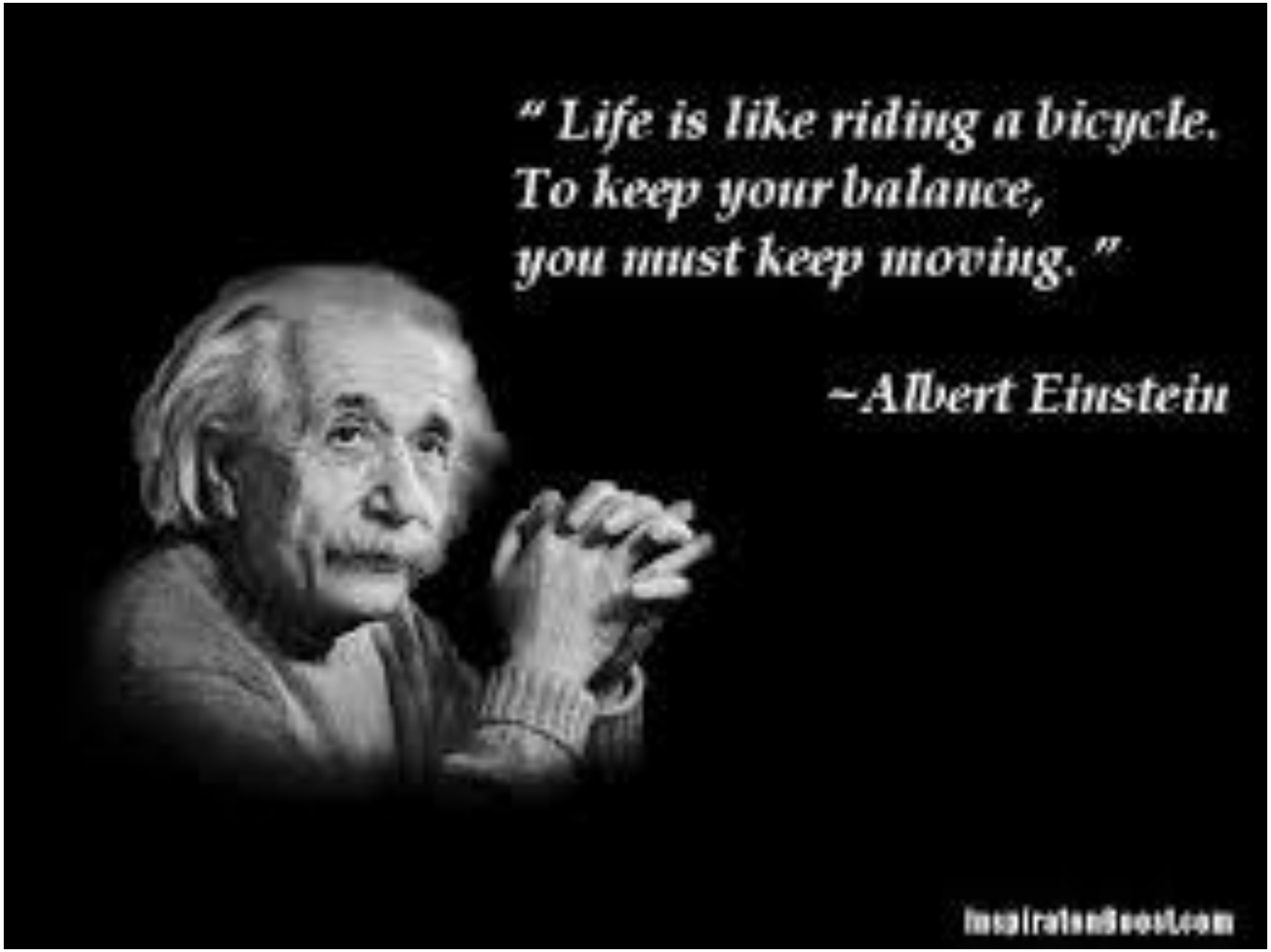
Theory of Relativity

UNIT I
Relativistic Mechanics
Lecture-6


" Life is like riding a bicycle.
To keep your balance,
you must keep moving. "
~Albert Einstein

MASS–ENERGY EQUIVALENCE

•Mass–energy equation is the most important and significant relationship derived by Einstein from the postulates of the special theory of relativity. This relation is also known as *scientific signature of Einstein. According to this relation:*

- *Total Energy E= Rest Mass Energy+ Relativistic KE*
- $E_{total} = m_0c^2 + (m - m_0)c^2$

MASS–ENERGY EQUIVALENCE CONTD…

- In order to derive this relation, let us consider a particle of rest mass m_0. It attains the mass m when acted upon by a force F to produce velocity u in the same direction. If the force F displaces the particle through a small distance dx, then the work done dw can be given as

$$dw = F \cdot dx$$

- This work done will appear as the increment in the kinetic energy as dK. Hence, we can write

$$dw = dK = F \cdot dx$$

- According to Newton's second law of motion

$$F = dp/ dt$$

where p is the momentum of the particle.

MASS–ENERGY EQUIVALENCE CONTD…

Now, $$F = \frac{d}{dt}(mv)$$

According to the theory of relativity, both m and v are variables in Eq. (1.40). Therefore,

$$F = m\frac{dv}{dt} + v\frac{dm}{dt}$$

$$dK = m\frac{dv}{dt}dx + v\frac{dm}{dt}dx$$

$$= m\frac{dx}{dt}dv + v\frac{dx}{dt}dm$$

$$dK = mv\,dv + v^2 dm$$

MASS–ENERGY EQUIVALENCE CONTD…

$$m = \frac{m_0}{\sqrt{1 - \frac{\upsilon^2}{c^2}}}$$

or $$m^2(c^2 - \upsilon^2) = m_0^2 c^2$$

Differentiating Eq. (1.42), we get

$$2m\,c^2\,dm - 2m\upsilon^2\,dm - 2m^2\,\upsilon d\upsilon = 0$$

or $$c^2\,dm = m\upsilon d\upsilon + \upsilon^2\,dm$$

$$dK = c^2\,dm$$

If the kinetic energy is changing from 0 to K due to change in its mass from m_0 to m, then

$$\int_0^K dK = \int_{m_0}^{m} c^2\,dm$$

Thus, $$K = c^2(m - m_0)$$

MASS–ENERGY EQUIVALENCE CONTD…

- From the expression of kinetic energy, it is clear that the increment in kinetic energy is due to the increment in the mass of the body on account of its relative motion.
- m_0c^2 is known as rest- mass energy, which may be considered as internal stored energy of the particle.
- Hence, the total energy of a moving particle is the sum of the kinetic energy and its energy at rest.

Total energy E = rest-mass energy + relativistic KE

$$= m_0c^2 + (m - m_0)\,c^2$$

$$E = mc^2$$

The above relation is the well-known *Einstein mass–energy relation*.

Experimental Verification of Mass Energy Equivalence

- Electron–positron pair annihilation and electron–positron pair production are the direct proofs of mass–energy equivalence.

$$e^{+} + e^{-} = \gamma$$

$$\text{(Particle having mass)} \rightarrow \text{(Photon having energy only)}$$

$$\gamma \rightarrow e^{+} + e^{-}$$

$$\text{(energy)} \rightarrow \text{(mass)}$$

- The mass–energy relation forms the basis for better understanding of nuclear phenomena like nuclear reactions such as fission and fusion.

- The tremendous energy produced during the explosion of atomic bombs is due to the conversion of mass into energy, according to this relation.

RELATIVISTIC RELATION BETWEEN ENERGY AND MOMENTUM

- The expression of the relativistic total energy of a particle moving with velocity v, is given as

$$E = mc^2$$

or

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where m_0 is the rest mass of the particle.

$$E^2\left(1 - \frac{v^2}{c^2}\right) = m_0^2 c^4$$

$$E^2 = \frac{E^2 v^2}{c^2} + m_0^2 c^4$$

$$E^2 = \frac{E^2 p^2}{m^2 c^2} + m_0^2 c^4$$

$$= p^2 c^2 + m_0^2 c^4$$

$$E = \sqrt{p^2 c^2 + m_0^2 c^4}$$

EXPRESSION OF RELATIVISTIC MOMENTUM

- From the expression of momentum in classical mechanics we know that

$$P=mv$$

- In the relativistic mechanics mass remains no longer constant but it vary with motion as $m = \frac{m_0}{\sqrt{1=\frac{v^2}{c^2}}}$

- The expression of relativistic momentum is given as

$$P= \frac{m_0 v}{\sqrt{1=\frac{v^2}{c^2}}}$$

Relativistic form of Newton's Second Law

- Classically the Newton's Second law can be given as
- F= $\frac{dp}{dt}$, Where p is the momentum.
- Using the expression of relativistic momentum we get
- $$F = \frac{d}{dt}\left[\frac{m_0 v}{\sqrt{1 = \frac{v^2}{c^2}}}\right]$$

$$F = \frac{dp}{dt} = \frac{d}{dt}\frac{m_0 \upsilon}{\sqrt{1-\frac{\upsilon^2}{c^2}}}$$

$$= m_0 \frac{d}{dt}\left[\upsilon\left(1-\frac{\upsilon^2}{c^2}\right)^{-1/2}\right]$$

$$= m_0 \frac{d\upsilon}{dt}\left(1-\frac{\upsilon^2}{c^2}\right)^{-3/2}$$

This is known as *relativistic form of Newton's second law.*

Massless particle and their explanation

- The particles having zero rest mass are known as massless particles.
- The existence of massless particle is beyond the scope of classical physics.
- But according to the relativistic mechanics, the particles with zero
- rest mass may exist.
- The energy and momentum of such a particle can be calculated with the expression of relativistic total energy E, which is given as

$$E = \sqrt{m_0^2 c^4 + p^2 c^2}$$

where p is the momentum of the particle.

For a massless particle, $m_0 = 0$

$\therefore$ $$E = pc$$

or $$p = \frac{E}{c}$$

Massless particle and their explanation Contd..

- Since p is also equal to mv = $\frac{E}{c^2}$v , because m = $\frac{E}{c^2}$
- Therefore, v = c, that is, the velocity of a massless particle is the same as that of light in free space.
- It indicates that a massless particle has mass as long as it is in motion.
- On being stopped, they cease to exist—they are either absorbed completely or changed into heat at the surface.
- Examples of massless particles are photons, neutrinos, and gravitons.
- A particle which has its rest mass zero and has its mass during its motion can be explained on the basis of the expression of variation of mass i.e. m=$m_0/\sqrt{1-v^2/c^2}$
- Actually, during the motion of the particle, the energy is converted into mass. It is also known as materialisation of energy, i.e., conversion of energy into matter.

summary

- Moving clocks run slow
- Moving objects appear shorter
- Moving object's mass increases

By a factor of γ

Assignments based on what we learnt in this lecture

- Obtain the expression of mass energy equivalence?
- Describe how the kinetic energy and total energy have different sense in relativistic and classical Mechanics .
- Give the experimental evidences in the support of Mass Energy equivalence.
- Establish the relativistic relation between energy and momentum.
- Obtain the relativistic form of Newton's Second Law.
- Explain the existence of Massless particles.